SM3 3.2: Rational Root Theorem

Memorize: Given a polynomial with lead coefficient q and constant p, the possible rational roots are given by $\pm\frac{factors\ of\ p}{factors\ of\ q}$.

Vocab: lead coefficient, constant, rational, synthetic division

For questions 1-3, state the possible rational roots:

1) $a(x) = x^4 + x^2 + 2x - 3$

$\pm 1, \pm 3$

2) $b(x) = 2x^2 - 5x + 3$

$\pm 1, \pm 3, \pm\frac{1}{2}, \pm\frac{3}{2}$

3) $c(x) = 4x^6 - x^5 + 3x^3 - 2x + 10$

$\pm 1, \pm 2, \pm 5, \pm 10, \pm\frac{1}{2}, \pm\frac{1}{4}, \pm\frac{5}{2}, \pm\frac{5}{4}$

For problems 4-12, find all of the zeros of each function and write in completely factored form:

4) $f(x) = x^3 - 3x - 2$

$x = \{-1, 2\}$
$f(x) = (x+1)^2(x-2)$

5) $g(x) = x^3 + x^2 - 80x - 300$

$x = \{-6, -5, 10\}$
$f(x) = (x+6)(x+5)(x-10)$

6) $h(x) = x^3 + 4x^2 + 3x$

$x = \{-3, -1, 0\}$
$f(x) = x(x+3)(x+1)$

7) $j(x) = 2x^3 - 15x^2 + 31x - 12$

$x = \left\{\frac{1}{2}, 3, 4\right\}$
$f(x) = (2x-1)(x-3)(x-4)$

8) $k(x) = 2x^3 - x^2 - 15x + 18$

$x = \left\{-3, \frac{3}{2}, 2\right\}$
$f(x) = (x+3)(2x-3)(x-2)$

9) $l(x) = 6x^3 - 5x^2 - 2x + 1$

$x = \left\{-\frac{1}{2}, \frac{1}{3}, 1\right\}$
$f(x) = (2x+1)(3x-1)(x-1)$

10) $m(x) = x^4 - 5x^2 - 36$

$x = \{-3, 3, -2i, 2i\}$
$f(x) = (x+3)(x-3)(x+2i)(x-2i)$

11) $n(x) = x^3 - 4x^2 + 6x - 4$

$x = \{2, 1-i, 1+i\}$
$f(x) = (x-2)(x-1+i)(x-1-i)$

12) $p(x) = x^3 - 5x^2 + 7x + 13$

$x = \{-1, 3-2i, 3+2i\}$
$f(x) = (x+1)(x-3+2i)(x-3-2i)$

For questions 13-15, find a third degree polynomial with rational coefficients that has the given roots.

13) $x = \{-3, 2, 1\}$

$x^3 - 7x + 6$

14) $x = \{2i, -2i, 3\}$

$x^3 - 3x^2 + 4x - 12$

15) $x = \{0, 5, -6\}$

$x^3 + x^2 - 30x$

16) $x = \{4, -3i\}$

$x^3 - 4x^2 + 9x - 36$